[0503] Consider now in more detail an individual sensor subsystem, i.e., the R1/T1 sensor subsystem shown in **FIG. 21**(*d*), the transmit array follows a section of a great circle that intersects the X axis and is rotated by an angle Θ about the X axis with respect to the equatorial plane. The tilt angle, say $\Theta=20°$, is less than the 23.5° latitude of the Tropic of Cancer.

[0504] In analyzing this array, let R be the radius of the hemisphere. Then the transmit array follows the following trajectory on the surface of the hemisphere.

$$x(s)=R\cdot\cos(\pi s/2)$$
$$y(s)=R\cdot\sin(\Theta)\cdot\sin(\pi s/2)$$
$$z(s)=R\cdot\cos(\Theta)\cdot\sin(\pi s/2)$$

[0505] The definitions used here for the x, y, and z directions are shown in **FIG. 21**(*d*). Similarly, the trajectory for the receive array is as follows.

$$x(s)=R\cdot\cos(\pi s/2)$$
$$y(s)=R\cdot\sin(\Theta)\cdot\sin(\pi s/2)$$
$$z(s)=-R\cdot\cos(\Theta)\cdot\sin(\pi s/2)$$

[0506] In these formulae, s is the path parameter which nominally varies from zero to one as the distances from the transducers increases. In this example, the array will start for a small positive value of s in order to make room for the finite sized transducer, and the array will end at a value of s slightly greater than one in order to provide the overlap between the sensor subsystem pairs discussed above.

[0507] Now consider the (θ,ϕ) coordinate system for the surface of the hemisphere defined by the following relations.

$$-\pi/2<\theta<\pi/2$$
$$0<\phi<\pi$$
$$x(\theta,\phi)=R\cdot\cos(\theta)\cdot\cos(\phi)$$
$$y(\theta,\phi)=R\cdot\cos(\theta)\cdot\sin(\phi)$$
$$z(\theta,\phi)=R\cdot\sin(\theta)$$

[0508] In terms of this coordinate system, the transmit array follows the trajectory:

$$\theta(s)=\arcsin(\cos(\Theta)\cdot\sin(\pi s/2))$$
$$\phi(s)=\arctan(\sin(\Theta)\cdot\tan(\pi s/2))$$

[0509] and the receive array follows the following trajectory:

$$\theta(s)=-\arcsin(\cos(\Theta)\cdot\sin(\pi s/2))$$
$$\phi(s)=\arctan(\sin(\Theta)\cdot\tan(\pi s/2))$$

[0510] The geodesic connecting the transmit and the receive arrays for the path parameter s is a segment of a line of longitude with respect to the z axis, namely the following section of a great circle.

$$-\arcsin(\cos(\Theta)\cdot\sin(\pi s/2))<\theta<\arcsin(\cos(\Theta)\cdot\sin(\pi s/2))$$
$$\phi=\arctan(\sin(\Theta)\cdot\tan(\pi s/2))$$

[0511] There are many options regarding choice of acoustic modes, and the particular spherical configuration does not alter the general principles of the invention. Let us consider in more detail the case in which the same acoustic mode propagates along both the transmit and receive arrays with group velocity V, while the mode, perhaps different, traversing the touch region has the group velocity V'. The delay time as a function of path parameter is given as follows.

$$T(s)=(R\cdot(\pi s/2))/V+2R\cdot\arcsin(\cos(\Theta)\cdot\sin(\pi s/2))/V'+(R\cdot(\pi s/2))/V$$

[0512] The delay time can also be expressed in terms of the coordinate ϕ of a touch which intercepts the acoustic path.

$$T(\phi)=(2R/V)\cdot\arctan(\tan(\phi)/\sin(\Theta))+2R\cdot\arcsin(\cos(\Theta)\cdot\sin(\arctan(\tan(\phi)/\sin(\Theta))))/V'$$

[0513] With this analytic expression, a look-up table may be calculated. Such a look-up table can be used in real-time microprocessor code to convert measured delay times of signal perturbations into the touch coordinate ϕ.

[0514] More generally, while explicit mathematical analysis may be able to determine a touch location on the surface, this analysis is not necessary in some cases. Rather, the transducers produce a set of outputs for a given touch condition, e.g., a location. By empirically determining a signature of this touch condition, the controller will be able to determine when this input condition subsequently occurs. Further, with a number of such conditions determined, an interpolation or statistical determination of the condition of an input determined, even if it does not identically correspond to a previously determined input condition. A lookup table is one way to store the data. Alternately, the data may be stored as coefficients of a compensation algorithm for mapping the input space into a desired output space.

[0515] The transducer pairs R1/T1 and R4/T4 provide complete coverage of the touch coordinate ϕ over the entire touch region.

[0516] Similarly R2/T2 and R5/T5 provide measurement of a touch coordinate u which is an equivalent to ϕ except that the polar axis, while still in the x-z plane, is rotated 60° with respect to the z axis. Likewise R6/T6 and R3/T3 provide a touch coordinate v which is an equivalent to ϕ rotated −60°. The three coordinates ϕ, u, and v provide redundant coverage of the dome sensor. In terms of x, y, and z coordinates, ϕ, u, and v are defined by the following relations.

$$\phi=\arctan(y/x)$$
$$u=\arctan\{y/[(1/2)x+(\sqrt{3}/2)z]\}$$
$$v=\arctan\{y/[(1/2)x-(\sqrt{3}/2)z]\}$$

[0517] The touch coordinate θ can be determined from ϕ, u as follows.

$$\theta(\phi, u)=\arctan[2\cot(u)\cdot\sin(\phi)/\sqrt{3}-\cos(\phi)/\sqrt{3}]$$

[0518] Likewise, the touch coordinate θ can be determined from ϕ, v as follows.

$$\theta(\phi, v)=-\arctan[2\cot(v)\cdot\sin(\phi)/\sqrt{3}-\cos(\phi)/\sqrt{3}]$$

[0519] If $\theta(\phi, u)$ and $\theta(\phi, v)$ agree, then (ϕ, u, v) form a self consistent triple of delay times; the meaning of self consistency of a triple is discussed above, e.g., in connection with item **2403** of **FIG. 24**(*a*). Thus, this sensor supports anti-shadowing and multiple-touch/redundancy-check algorithms.

[0520] If we define $\delta\theta=\theta$, $\delta\phi=\phi-\pi/2$, $\delta u=u-\pi/2$, and $\delta v=v-\pi 2$, then the top of the dome sensor corresponds to the values $\delta\theta=0$, $\delta\phi=0$, $\delta u=0$, and $\delta v=0$. Taylor expanding the above relations about the top of the sensor gives the following approximate relations.

$$\delta\theta=\delta\phi/\sqrt{3}-2\delta u/\sqrt{3}$$
$$\delta\theta=-\delta\phi/\sqrt{3}+2\delta v/\sqrt{3}$$
$$\delta\theta=(-\delta u+\delta v)/\sqrt{3}$$
$$\delta\phi=\delta u+\delta v$$